

### QUIZ 3

*Instructions: Please write your name at the top of the page. Show all your work and explain your reasoning unless otherwise indicated.*

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- (1) (5 points). Let  $\phi : R \rightarrow S$  be a ring homomorphism. Carefully prove that the induced map on spectra  $\phi^\# : \text{Spec}(S) \rightarrow \text{Spec}(R)$  is continuous in the Zariski topology.

(2) (5 points). Let  $R$  be a ring and let  $f_1, f_2, \dots, f_t \in R$  be non-zero elements such that  $\{D(f_i)\}_{i=1}^t$  form an open cover of  $\text{Spec}(R)$ . Carefully prove that  $R$  is reduced if and only if  $R[\frac{1}{f_i}]$  is reduced for all  $i$ .