

QUIZ 4

Instructions: Please write your name at the top of the page. Show all your work and explain your reasoning unless otherwise indicated.

- (a) (5 points). Consider the linear map $\pi : \mathbb{Q}^3 \rightarrow \mathbb{Q}^3$ given by the matrix A below. Find a basis of $\ker(\pi^\vee)$.

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \end{pmatrix}.$$

- (b) (5 points). Let $\phi, \psi : \mathbb{Q}^2 \rightarrow \mathbb{Q}^2$ be the linear maps given by the matrices:

$$[\phi] = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad [\psi] = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}.$$

Compute the matrix of $\phi \otimes \psi$ in the basis $e_1 \otimes e_1, e_1 \otimes e_2, e_2 \otimes e_1, e_2 \otimes e_2$. Write $(\phi \otimes \psi)(e_1 \otimes e_1 + 2e_1 \otimes e_2 + e_2 \otimes e_1 - e_2 \otimes e_2)$ as a single pure tensor.