

QUIZ 2

Instructions: Please write your name at the top of the page. Show all your work and explain your reasoning unless otherwise indicated.

- (a) (5 points). Consider the map ϕ defined below, which on Quiz 1 you prove was a homomorphism of $\mathbb{Z}$ -modules:

$$\phi: \mathbb{Z} \longrightarrow \mathbb{Z}/12\mathbb{Z} \quad \text{given by} \quad n \longmapsto \overline{8n}.$$

Write down explicitly the isomorphism supplied by the first isomorphism theorem.

- (b) (5 points) Let $V = \mathbb{K}^2$, with standard basis e_1, e_2 and define a $\mathbb{K}$ -linear map $T: V \rightarrow V$ by $T(e_1) = 0$ and $T(e_2) = e_1$. Let V_T denote the $\mathbb{K}[x]$ -module in which x acts on V by T . Compute $(1 + x + x^2)e_2$ in V_T .